Accelerating the convergence in the identification of PV cells parameters

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***Abstract*—The current trend in energy sources is towards the large-scale use of small-scale photovoltaic (PV) systems for local energy supply. The today's common commercial PV systems today include PV cells of various manufactures and origins based mostly on a large-area p-n junction made of silicon. The characteristic equation describes the non-linear dependence between current and voltage and the parameters values of the characteristic equation define the working regime of the PV system. Parameters identification involves a series of iterations in which the sum of squared errors is minimized (and thus the chance of observation is maximized). A large number of iterations are required if the optimization follows the usual course from the solution proposed by the vertical offsets to the perpendicular offsets. In the present work, an intermediate solution is used to speed up the convergence.**

***Index Terms*—PV cells, parameters estimation, perpendicular offsets, nonlinear regression**

I. Introduction

After many centuries of exploitation of polluting energy sources that started with coal [1], continued with oil [2] and more recently with uranium [3], today we are reorienting towards renewable energy. However, the first studies on solar energy conversion date back to 1839 [4] and the first PV system was patented in 1905 [5].

A PV cell is a specialized semiconductor component that converts visible (VIS), infrared (IR) or ultraviolet (UV) radiation into electrical current [6]. The technology to produce a commercial PV cell is based on the recrystallization of silicon with hydrochloric acid and copper [7].

Natural counterparts of PV cells are the cholorphyls which converts the solar energy with about 1% [8], with a peak efficiency of about 3% and a theoretical efficiency of 9% [9]. A record for solar cell efficiency, namely 47.1%, was recently achieved by using multi-junction concentrator [10]. However, there is a long way to go in order transfer this peak performance into mass production, since commercial solar cells may have just about 10% efficiency [11]. In the same time, the performance of PV cells can be further increased by continuous alignment [12].

PV is approximated with an idealized system consisting of a series of active and passive components (see §2.1 in [6] and references therein). Lambert function [13] can be used then to derive a pseudo-explicit expression of current as a function of voltage (or vice versa). However, explicitly the function that expresses voltage as a function of current (or vice versa) requires a series of successive approximations formalized with the help of the function proposed by Lambert, $z = \mathrm{W}(z)\exp(\mathrm{W}(z))$. Alternate approach is to use explicit equations approximating current vs. voltage (or vice versa) dependence (see §2.3 and §2.6 in [6]). However it may be, the model parameters depends on the construction of the PV cell and requires identification.

Series of paired (current, voltage) data are collected from the working regime of the PV cell. In this context, both measurements are affected by experimental error and the use of the vertical offsets model in estimating the regression parameters is not appropriate. Perpendicular offsets balances both current and voltage measurement errors. However, the number of iterations from the solution proposed by the vertical offsets approach to the solution proposed by the perpendicular offsets is huge due to the fact that solving the problem requires doing an optimization inside an optimization (see §2.7 and §3.1 in [6]). The alternative to provide an intermediate solution is explored here.

II. Material and Method

In [6] work the full approach of perpendicular offsets has been employed to identify the parameters for two nonlinear regressions. However, a huge number of iterations were required to arrive to the optimal solutions (Tab. I).

TABLE I
Solutions proposed by the vertical and perpendicular offsets for the coefficients and iterations till convergence.

$f(x;c)$	coeffs	$\vert$ offsets	$\perp$ offsets	iterations
$\frac{c_0 + c_1 x + c_2 x^2}{c_3 + x}$	c_0	−3350	−3349.81	2,482,076,218
	c_1	2689	2689.40	
	c_2	−476	−475.164	
	c_3	−1.93	−1.92823	
$c_1 - \exp(c_2 + c_3 \ln(x))$	c_1	1.83	1.82622	1,072,445,080
	c_2	−22	−22.2914	
	c_3	3.07	3.11345	

$f(x;c)$ & coeffs: the nonlinear model & its parameters (eqs. 11 & 12 in [6])
$\vert$ & $\perp$ offsets: initial (guess) & final (optimized) values (tabs. 2 & 3 in [6])
iterations: from $\vert$ to $\perp$ (using Alg. 3 from [6]; given in Fig. 13 in [6])

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A simplified approach may provide the acceleration of the convergence. For convenience the same data given in [6] in Tab. 1 is used here. Let's consider for convenience the function $y = f(x; c)$ expressing analytically the voltage (V) as function of amperes (A). The equation of the tangent is given in Eq. (1).

$$y = f(z_i; c) + (x - z_i)f'(z_i; c), \quad (1)$$

where z_i is the ordinate position of the contact point between the tangent and the curve (the equation of the tangent to $y = f(x; c)$ in $x = z_i$), and $f'(x; c)$ is the function derivative ($f'(x; c) = \mathrm{d}f(x; c)/\mathrm{d}x$).

The equation of the normal to the $y = f(x)$ curve is given in Eq. (2).

$$y = f(z_i; c) - (x - z_i)/f'(z_i; c) \quad (2)$$

The contact point is $(z_i, f(z_i; c))$ and the normal intersects the observation point $((x_i, y_i))$, so (using Eq. (2)):

$$y_i = f(z_i; c) - (x_i - z_i)/f'(z_i; c) \quad (3)$$

The numeric value of z_i is to be obtained from Eq. (3) by root finding. With the value of z_i the value of the perpendicular offset d_i is given in Eq. (4).

$$d_i^2 = (z_i - x_i)^2 + (f(z_i; c) - y_i)^2 \quad (4)$$

Alternately (to the root finding of Eq. (3)), in most of the cases (for smooth variations) is enough to find the z_i for which d_i is minimum, and this is the exact approach of using perpendicular offsets used in [6].

For each observed pair $((x_i, y_i)$ from a set of n) the vertical offset is $|y_i - f(x_i; c)|$ while the horizontal is $|x_i - f^{-1}(y_i; c)|$ such that the height (h_i) of the triangle having these two offsets as legs is given by Eq. (5).

$$h_i^2 = \frac{(f(x_i; c) - y_i)^2(f^{-1}(y_i; c) - x_i)^2}{(f(x_i; c) - y_i)^2 + (f^{-1}(y_i; c) - x_i)^2} \quad (5)$$

At small departures this height (Eq. 5) approximates the length of the perpendicular offset (Eq. 4, $h_i \approx d_i$, Fig. 1).

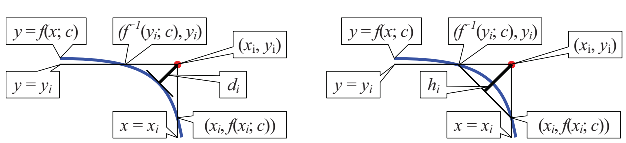

Fig. 1. Perpendicular (d_i) offset and its approximation (h_i)

We use the Eq. (5) approximation to obtain a new guess for the model parameters c here. It should be noted that when the inverse of the function (f^{-1} in Eq. (5)) is available and explicit, the use of Eq. (5) does not requires root finding (solving of Eq. (3)) or minimization (of Eq. (4)). This is the main advantage of the proposed method and the hope is that it will dramatically reduces the number of iterations till optimum values (accelerated convergence).

If MINIMIZE(g, c) solves an optimization problem where g is the objective function to be minimized and c are the unknown coefficients (to be found) on which the value of the objective function depends then the iteration to the optimal values of the parameters (c) for $\Sigma h_i^2 \rightarrow$ min. is as follows:

- $c \leftarrow$ optimum values from minimizing the sums of the residuals with classical vertical offsets ($\sum_{i=1}^{m}(y_i - f(x_i; c))^2$);
- $g(c) \leftarrow \sum_{i=1}^{m} h_i^2$, with h_i^2 from Eq. (5);
- MINIMIZE(g, c).

The second stage of the optimization is to use the new values of the coefficients as initial values for the optimization minimizing Σd_i^2:

- $c \leftarrow$ optimum values from minimizing the sums of h_i^2 (Eq. 5);
- $k(c) \leftarrow \sum_{i=1}^{m} d_i^2$ with d_i^2 from Eq. (4);
- MINIMIZE(k, c).

III. RESULTS AND DISCUSSION

The intermediary values of the model parrameters corresponding to $\Sigma h_i^2 \rightarrow$ min. reduced considerably the number of iterations. Tab. II contains the descriptive info.

TABLE II
SOLUTIONS PROPOSED BY THE EQ. 4 AND EQ. 5 AND ITERATIONS TILL CONVERGENCE.

$f(x; c)$	coeffs	Eq. 4	Eq. 5	iterations
$\frac{c_0 + c_1 x + c_2 x^2}{c_3 + x}$	c_0	−3349.81	−3349.81	8,930,636
	c_1	2689.41	2689.41	+
	c_2	−475.156	−475.151	53,146,935
	c_3	−1.92821	−1.92819	
$c_1 - \exp(c_2 + c_3 \ln(x))$	c_1	1.82635	1.82628	4,739,635
	c_2	−22.2710	−22.2815	+
	c_3	3.11055	3.11204	73,344,752

$f(x; c)$ & coeffs: the nonlinear model & its parameters
Eq. 4 & 5: parameter values for $\Sigma d_i^2 \rightarrow$ min. & $\Sigma h_i^2 \rightarrow$ min.
iterations: from Tab. I | offsets to Eq. 4 & from Eq. 4 to Eq. 5

Information listed in Tab. II reveals that after the first stage of the optimization (when Σh_i^2 is minimum) the values of the parameters corresponding to $\Sigma d_i^2 \rightarrow$ min. are nearly to their optimal values. For the model with four parameters the changes are at the last digit: c_0(Eq. 4) = c_0(Eq. 5), c_1(Eq. 4) = c_1(Eq. 5), c_2(Eq. 4) = c_2(Eq. 5) − 0.005, c_3(Eq. 4) = c_3(Eq. 5) − 0.00002. For the model with three parameters the changes (from Tab. II Eq. 4 to Tab. II Eq. 5) even if appears at more digits are small, all being below 5% of the changes from the initial (Tab. I | offset) to the intermediary (Tab. II Eq. 4) values. The variation in the first stage of the optimization is smooth (see Figs. 2 - 8). Overall, also in the first stage of the optimization (and even more pregnant in the second, see Tab. II) the changes in the parameters values are small. The explanation for this fact is that the perpendicular offsets are near as long as the vertical ones, the length of the horizontal one being much longer in comparison. As consequence, it must be that the experimental error of produced with the use of the voltmeter was much smaller than the experimental error of produced with the use of the ammeter. Similar behaviour is found for the evolution of all parameters.

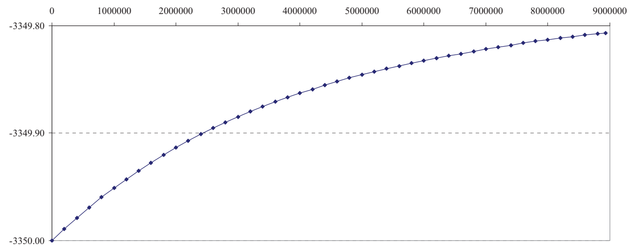

Fig. 2. Change with iteration for the parameter c_0 of the 4 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

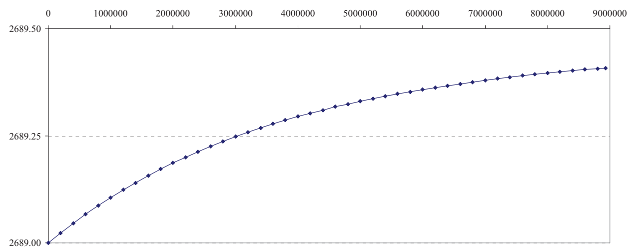

Fig. 3. Change with iteration for the parameter c_1 of the 4 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

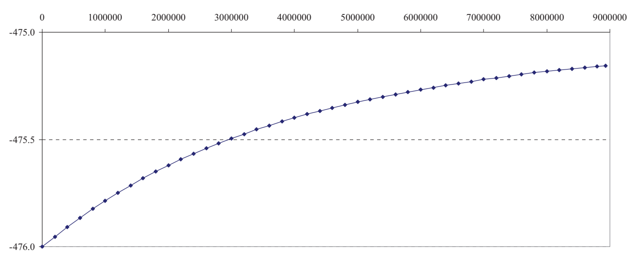

Fig. 4. Change with iteration for the parameter c_2 of the 4 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

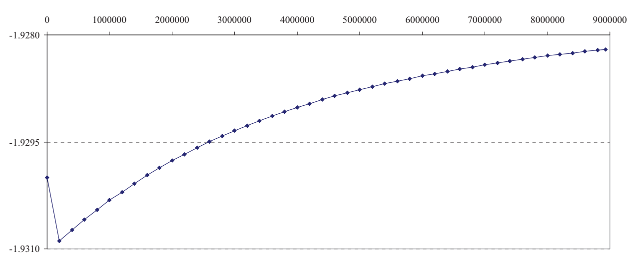

Fig. 5. Change with iteration for the parameter c_3 of the 4 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

Even if the changes seems unsignificant, since both experimental measurements - of current and of voltage - are subjected with the same chance to contain experimental error the appropriate approach is to estimate the parameters using the perpendicular offsets.

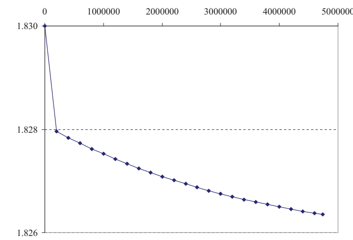

Fig. 6. Change with iteration for the parameter c_1 of the 3 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

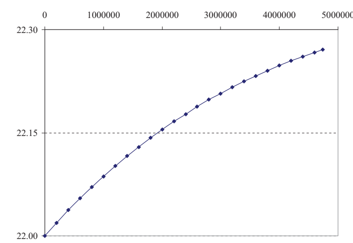

Fig. 7. Change with iteration for the parameter c_2 of the 3 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

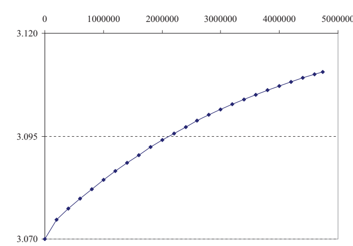

Fig. 8. Change with iteration for the parameter c_3 of the 3 parameters model in the first stage of the optimization (to $\Sigma h_i^2 \rightarrow$ min.)

The convergence is fast by using intermediate optimum values. The number of iterations is reduced significantly: about 40 times for the model with 4 parameters and about 14 times for the model with 3 parameters (see the number of iterations in Tab. I vs. the number of iterations in Tab. II).

IV. Conclusion

For sets of paired data perpendicular offsets approach should be used. If the perpendicular offsets approch used for nonlinear regression take a considerable amount of iterations to optimal values of the parameters, convergence is accelerated when approximate perpendicular offsets are intermediary calculated.

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